The Distributional Impacts of Transportation Policies: A Research Design for the Case of Roadway Tolls

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Abstract

Highway congestion in North America is widely responsible for polluting air, increasing noise, and wasting time spent in traffic, largely because drivers do not pay the full marginal cost of their own traveling decisions. Economists agree that a well-designed congestion pricing system can correct for these externalities, but there is widespread political and public resistance to such a plan, possibly due to disagreements about its potential distributional effects.

This paper documents a research design for examining the distributional effects of transportation policies, using a bridge toll as a case study. The methodology employs the mode choice component of a conventional travel model, implemented as a binomial logit. To measure individual welfare, a compensating variation estimator that allows income to have an effect on choice is used, where the marginal utility of money is allowed to vary by income level; this helps avoid understating the welfare effects on low-income individuals. The estimated individual welfare distributions for all individuals can then be compared for the *no-toll* and *with-toll* scenarios using a collection of quantitative assessment tools, such as: divergence measures to distinguish any redistributive effect; relative distributions to characterize the shape of the redistributive effects and identify income ranges that are affected differently; and polarization indices to summarize whether the effect is progressive or regressive. By combining several innovative approaches that have only been used separately in the literature, the proposed approach informs the discourse on roadway pricing by examining welfare effects across the entire range a distribution, without discarding important distributional information at any stage of the analysis.

1 Introduction

Highway congestion in North America is widely responsible for polluting air, increasing noise, and wasting time spent in traffic, largely because drivers do not pay the full marginal cost of their own traveling decisions. Economists agree that a congestion pricing system can correct for these externalities if it can exactly offset the difference between the total marginal cost of travelers' decisions and the direct costs they currently pay. However, there is widespread political and public resistance to such plans, possibly due to potentially detrimental effects on income equality, i.e. equity.

The chief argument against tolls is that lower-income individuals are less likely to have enough travel patter flexibility to avoid peak-hour tolls, while the counterargument is that they are also more likely to already be taking transit anyway. Many theoretical studies (Arnott et al. 1994, Glazer & Niskanen 2000, Small 1983) have examined this question, finding somewhat conflicting results. While some quantitative studies of welfare effects using implemented travel demand models exist (Eliasson & Mattsson 2005, Kalmanje & Kockelman 2004) they are not conclusive with respect to distributional questions. In these studies, equity has been examined only with respect to broad categorical income levels, which may fail to identify important features of a congestion pricing plan's effective redistribution of welfare. Moreover, these studies have, for the most part, used welfare measures that are insensitive to income effects, rendering the equity conclusions suspect.

This paper documents a research design for examining the distributional effects of transportation policies, using roadway tolls as a case study, for the purpose of contributing to the discourse surrounding the distributional effects of a roadway tolling policy. The methodology employs the mode choice component of a conventional travel model, implemented as a binomial logit. As a measure of individual welfare, a compensating variation can be computed using a procedure that allows income to have an effect on choice, and for the marginal utility of money to vary by income level, which helps to avoid understating the welfare effects on low-income individuals. The estimated individual welfare distributions for all individuals can then be compared for the *no-toll* and *with-toll* scenarios using a collection of quantitative assessment tools, such as: divergence measures to distinguish any redistributive effect; relative distributions to characterize the shape of the redistributive effects and identify income ranges that are affected differently; and polarization indices to summarize whether the effect is progressive or regressive.

This approach will be especially informative on distributional effects because it combines several innovative techniques of examining welfare effects across the entire range a distribution. First, by using a random utility modeling approach, we allow the unobserved portion of the utility estimates for individuals who change mode between auto and bus to vary across a random distribution. Second, by using a transcendental logarithmic function for the observed utility, we can accommodate flexible substitution patterns between the two major commodities, time and money. Third, by applying a compensating variations measure that incorporates income effects, we allow the marginal utility of money to vary by income level, and we can avoid understating the effects of a policy on lower-income individuals. Finally, by applying a variety of distributional analysis tools, we can interpret changes in the welfare distributions in ways that directly inform some specific policy questions, such as: What parts of the income distribution will benefit from the plan, and what part will lose, if any? How does the current travel pattern (e.g. transit usage) affect the distribution of costs and benefits? What is the actual benefit received by roadway users, relative to the perceived cost? How many people gain from the system, versus how many will lose, and hence how might this balance affect the plan's popularity? As available, early quantitative results from a Seattle case study may also be presented.

This paper continues in Section 2 with some background on transportation policy welfare analysis and the discourse on congestion pricing's welfare effects. Next, in Section 3, I present the research question and discuss how that question can be operationalized into a set of specific hypotheses. In Section 4, I define the behavioral model that I use to represent individual choice behavior for the choice of mode-to-work, and in Section 5, I present the conditions of the case study, which involves a new bridge toll near Seattle, WA. I then describe a methodology for analyzing the results of a simulated policy, first in terms of estimating individual welfare in Section 6, then in using some quantitative tools to interpret the distributions of welfare estimates for an entire population in Section 7. In Section 8, I link the quantitative tools to the hypotheses by defining explicit hypothesis tests. Finally, I make some preliminary concluding remarks in Section 9.

2 Background

To place this approach in context, I focus on two related channels of research: a) theoretical approaches to quantitatively evaluating welfare, giving particular emphasis to methods of evaluating equity; and b) the particular welfare and equity effects of one kind of transportation policy, *roadway pricing*, where much of the literature focuses particularly on congestion pricing.

2.1 Theoretical Approaches to Welfare Analysis

The primary difficulty in evaluating welfare associated with a transportation policy is in defining what distributions of welfare would be preferable to others. Some strategies can be found in several theories of justice and social welfare.

Utilitarian Theory By far, the most common approach to transportation policy welfare analysis is to quantify the aggregate, or total, welfare change to society at large:

$$w_{\mathcal{P}} = \sum_{i \in \mathcal{P}} w_i, \tag{1}$$

where $w_{\mathcal{P}}$ is population $\mathcal{P}$'s overall welfare and w_i is the welfare of individual i. This approach is the Utilitarian approach to just decision-making, which considers all welfare of all individuals in a society to have equal weight. In other words, if you give a dollar to a rich person, or give a dollar to a poor person, it has the same effect on society as a whole. Benefit-cost analyses are essentially utilitarian welfare analyses because they treat all bearers of costs and benefits equally in summation-style aggregation. The utilitarian approach also has linkages to Pareto-optimal economics, in that if we can also assume there is a mechanism by which the gainers from a policy are able to compensate the losers for their loss, then a Utilitarian-improving policy is also Pareto-improving, since *after compensations*, no one will be worse off than before, and at least some will be better off than before.

Rawlsian Theory Many philosophers and economists have illuminated the inability of a Utilitarian or a Pareto-optimal welfare analysis to account for equity aspects of policy choices; most importantly, these approaches disregard *who* benefits from a policy, so even when there are no losers, it is taken as inconsequential whether the gainers are the high-welfare or low-welfare members of society. Rawls (1971) argues that if we were to truly disassociate ourselves with our place in society (i.e. to take on Rawls' "veil of ignorance"), then we would be more concerned with improving the well-being of the low end of society, a notion captured in his “difference principle”. In this principle, one would seek the policy that ultimately does the best for the worst-off member of society:

$$w_{\mathcal{P}} = \min_{i \in \mathcal{P}} w_i. \tag{2}$$

Social Welfare Functions Some welfare economists have argued that Rawls overemphasizes the low-end of the distribution, to the detriment of the distribution among the remainder of the welfare range. Bergson (1938) proposed, instead, the use of a weighted sum of individual welfares, known as a *social welfare function*:

$$w_{\mathcal{P}} = \sum_{i \in \mathcal{P}} \gamma_i w_i, \tag{3}$$

where γ_i is the weight attributed to person i. Cowell (1977) expanded further on the specification of social welfare functions, but the important point is that lower-welfare individuals should always be weighted greater than higher-welfare individuals, hence there is a negative monotonic relationship between w_i and γ_i.

In practice, the difference principle and the social welfare function are difficult to implement. Rawls’s difference principle is rarely applied because it requires that we know about the worst-off member of society, but with normal social survey methods, this person is often unobservable. The Social Welfare Function approach is also demanding because there is no clear method of assigning the values of the social weights, γ_i. However, several other equity evaluation methods have emerged as common in real-world welfare analyses, including welfare percentile comparisons, poverty-line comparisons, and a Lorenz Curve/Gini Coefficient analysis. These all share Rawls’ goal of capturing effects at the bottom of the welfare distribution, but are simpler to implement using available data.

The first method, percentile group comparisons, attempts to measure changes across the entire welfare spectrum at a resolution that is detailed enough to capture the dominant trends, but coarse enough to be represented in categorical form. The second is a simpler approach, delineating only two categories that are meant to represent those who have a welfare sufficient to meet basic needs, and those who do not. In the poverty line approach, we are concerned with size of each group, and we seek to move people under the poverty line to over the poverty line. The third and final method is not categorical, but rather seeks to summarize the degree of inequality in graphical form of a Lorenz Curve (Lorenz 1905) and in a single summary number, the Gini Coefficient (Gini 1912). While the Coefficient originated as a convenient summary of the Lorenz Curve, Cowell (1977) also showed it to exhibit several desirable behavioral properties as a measure of inequality.

Given the variety of theories and methods above, it should be clear that there is no consensus on how to approach a quantitative equity study. Instead of choosing a single approach among these, it seems preferable to choose an approach in each case that is responsive specifically to a policy question at hand.

2.2 Welfare Effects of Road Tolls

In congestion pricing, we see one of the most actively discussed strategies for improving social welfare using a transportation policy. Congestion pricing has long been proposed as a simple means of improving the economic efficiency of roadways, and hence increasing aggregate social welfare. According to prevailing theories of roadway dynamics, as traffic demand approaches a roadway’s capacity, overall traffic speeds decrease. The resulting increased travel times are externalities of individual motorists' decisions to take a particular roadway, since their decisions neglect the travel time costs on other users. Consequently, the user-equilibrium traffic distribution is distinctly less efficient than the globally optimal traffic distribution.

Congestion pricing systems have been proposed to charge motorists for the difference between the user cost and the full social cost of the decision to take a particular roadway, thereby bringing the user equilibrium and the global optimum into convergence. The literature largely agrees on the efficiency benefits of a theoretical first-best congestion pricing system, as well as of a more feasible second-best system that is subject to various unavoidable constraints to how the transportation system is priced. However, the theoretical arguments on the equity impacts of congestion have been varied.

2.2.1 Pricing Schemes

Roadway pricing theory originated with Pigou (1952), who used a simple two-road example to demonstrate the concept of an externality caused by a capacity constraint on roadway flow, along with a charge on the constrained road that would maximize total utilitarian social welfare . Vickrey (1969) later reformulated the problem as a "bottleneck" constraint with a time dimension, allowing the theory to endogenize the costs associated with sub-optimal departure and arrival times.

Both formulations continue to be found in theoretical research, although for most practical applications in travel demand modeling, the time dimension is not explicitly represented, making the Vickrey model impossible to apply in these contexts. In both of these models, it is well understood that under congested conditions, an additional roadway user pays less (in the form of her own travel time costs) than the social marginal costs she incurs (in the form of increased travel time for all of the roadway's users at that time). When we consider implementing roadway pricing scheme, an optimal, or "first-best" pricing scheme, is one in which each roadway user pays exactly the marginal cost of his trip on every component of the transportation system at the time of day he passes through it.

The implementation of a first-best charging scheme would necessitate continuously variable tolls that are matched to specific locations along the entire transportation network, something that would be nearly impossible to implement. The alternative pricing theory is the "theory of the second best", which allows structural restrictions on how the toll system is implemented. Second-best toll systems are usually characterized by the placement of tolls only on either a cordon around a central city or on other key facilities, rather than on the entire roadway network. The level of the toll is either fixed, or it varies by steps, usually at specified times of day, rather than in response to toll levels. The tolls system considered in this proposal falls into this second-best pricing category.

2.2.2 Equity Arguments

The strongest criticisms of congestion pricing have come from those who argue that it would have regressive effects, in other words, that it would benefit higher income groups more than lower income groups, or even benefit higher income groups at the expense of lower income groups.

Regressive Effects In several theoretical studies using Vickrey's dynamic congestion model with commuters that are heterogeneous in their travel time and schedule delay costs, Arnott et al. (1993, 1994, 1998) compared welfare effects on discrete income groups, finding congestion pricing to be regressive, and specifically, that the benefits favor those that value travel time the most and value taking the trip the most. These results confirmed several earlier studies' findings suggesting regressive effects (Layard 1977, Cohen 1987, Evans 1992) when using static congestion models. More recent work (Raux & Souche 2004) has also confirmed these results.

For the most part, these studies disregarded the redistributive effects of toll revenue usage structure, which could significantly alter the equity results (Small 1983). Small's study found that benefits under several refund schemes could benefit all income levels, although once travel time benefits are accounted for (which may be equal across incomes), higher-income groups will continue to enjoy the greatest benefit, due to their higher value of time.

Progressive Effects As summarized by Eliasson & Mattsson (2005), most arguments regarding the equity effects of congestion pricing have supported the notions that the positive and the negative effects of congestion pricing favor either higher-income individuals or lower-income individuals. Progressive effects have been supported by Foster (1974), arguing that higher-income individuals are those most likely to be affected by a toll, since they choose to drive private cars more often and they tend to have a home-to-work trip originating in the suburbs and ending in the central city. This overall result was borne out in a study by Santos & Rojey (2004), although they note that there are a small number of individual cases where low-income payers would experience regressive effects.

Eliasson & Mattsson (2005) argue that the variability in the above literature can be explained by differences in two key factors: the initial set of travel patterns, and the scheme used for redistributing collected tolls. This latter point is supported by the theoretical arguments set forth by Mayeres & Proost (1997). Regardless, Giuliano (1994) maintains that redistribution cannot resolve all equity concerns due to individual- and household-level variation in flexibility with respect to alternate modes or work schedules.

Eliasson & Mattsson (2005) also note that the magnitudes of differences in effects for different income levels, as observed in past studies, is far surpassed by the magnitudes of incomes themselves, implying that whatever progressivity or regressivity exists would be quite small. Instead, they approach the question of equity by comparing average net effects on discrete or discretized demographic dimensions, regardless of the exact initial income level. These dimensions included gender, employment status, family type, geographic area of residence, and three income groups that represent equal proportions of the sample.

3 Research Question

Fundamentally, the question I am investigating is:

> *In what ways do transportation policies change the distribution of welfare among a population?*

To operationalize this question, I am considering individual persons as my unit of analysis in the study "population" and am limiting my scope to working adults, chiefly because this makes my study well suited to available data. By "welfare", I refer to the abstract sense that incorporates not only explicit monetary wealth, but also the implicit aspects of well-being that manifest themselves through individual preferences, as conceptualized by random utility theory. Finally, I take "distribution" to mean a collection of distinct values for the study population, quantitatively represented by a vector. If, then, we conceive of these welfare levels as having been drawn from some underlying smooth probability density function, then the analysis of the welfare distribution can follow analogously to a probability distribution. This analogy will be useful when employing some distributional analysis tools to interpret the results.

3.1 Hypotheses

The fundamental research question above is too general to be used in developing specific hypotheses. Moreover, it is only loosely connected to relevant policy issues. Instead, I outline here a set of hypotheses that have been investigated in past studies or are of interest to the current discourse. Here, I phrase my hypotheses as questions, but in Section 8, I will present them again more formally, as asserted null-hypothesis statements.

Hypothesis 1. *Does the policy make a substantial difference?* The most basic question we might ask is whether the policy has any effect at all, without narrowing the inquiry to a particular dimension of change. This will be especially informative in cases where a policy shows no specific kinds of effects with regard to the other questions below, and yet still there seems to be some kind of change. In such a case, we might conclude that the set of hypotheses that follow is inadequate in breadth to capture the relevant kinds of changes in a welfare distribution.

Hypothesis 2. *Does the policy make an overall improvement?* Following the utilitarian approach, this question regards the change in location of a welfare distribution: does it, overall, increase in magnitude, translating to an overall improvement? The conclusion should be consistent with the result of a conventional benefit-cost analysis.

Hypothesis 3. *Does the policy reduce (or increase) the number of individuals under the poverty line?* Here we examine the policy's effect on poverty by questioning the number of people above or below the poverty line.

Hypothesis 4. *Is the policy regressive (or progressive)?* Now we begin to examine the equity dimension of the policy. By referencing *progressive* versus *regressive* effects, this question adopts the terminology of the Lorenz Curve and Gini Coefficient.

Hypothesis 5. *Does the policy polarize (or depolarize) the spread of welfare levels?* We can also examine equity effects with reference to the degree of *polarization* exhibited. This approach is distinct from the regressive-progressive approach, in that here, we explicitly remove the aggregate effects (i.e. an overall improvement or worsening) and examine only the change in shape of the welfare distribution.

Hypothesis 6. *Does the policy have different effects on distinct welfare groups?* Here, we divide the population into several welfare-percentile ranges. This is the categorical approach taken by many conventional equity studies. It is distinct in that it relies on *a priori* notions of how groups should be defined.

Hypothesis 7. *If the policy has heterogenous effects on the welfare distribution, then which parts gain or lose?* This question requires more than a simple binary response. Instead, we now look at the specific gains and losses exhibited across the range of welfare levels. Conventionally, this has been accomplished by separately quantifying the effects for distinct welfare-level ranges, but such a categorization is not necessarily required.

Hypothesis 8. *What is the balance between those with a net gain and those with a net loss?* I end with a question rooted in the pragmatic issue of whether a proposed policy would pass a general referendum, provided that all individuals would vote equally and decide rationally based on their own individual welfare levels. To answer this, the relevant question is whether a majority gains, or a majority loses, from the policy. This approach may help inform us on whether a policy may be considered "popular" or not.

4 Behavioral Model

To test the set of hypotheses, I first employ a random utility theoretic perspective on travel choice behavior. Consider a population $\mathcal{P}$ of individuals, each choosing from the same set of discrete, mutually exclusive alternatives $\mathcal{A}$. We assume that each individual chooses the alternative that maximizes his or her own utility. Therefore, we can express an individual's utility as:

$$u_i \equiv \max_{k \in \mathcal{A}} \{u_{i,k}\} \quad \forall \quad i \in \mathcal{P}, \tag{4}$$

where $u_{i,k}$ is the utility that individual i derives from alternative k. We denote whether an individual actually chose an alternative by:

$$\delta_{i,k} = \mathcal{I}\left[u_{i,k} > u_{i,l} \; \forall \; l \neq k\right] = \begin{cases} 1 & \text{if } u_{i,k} > u_{i,l} \; \forall \; l \neq k \\ 0 & \text{otw} \end{cases} \quad \forall \quad i \in \mathcal{P}; k, l \in \mathcal{A}, \tag{5}$$

where $\delta_{i,k}$ is a binary indicator of whether i chooses alternative k, and $\mathcal{I}[\cdot]$ is the indicator function. To ensure that exactly one alternative is chosen by each individual, we also say:

$$\sum_{k \in \mathcal{A}} \delta_{i,k} = 1 \quad \forall \quad i \in \mathcal{P}. \tag{6}$$

We as analysts can observe the matrix of actual choices $\boldsymbol{\delta}$ for a survey sample, but we can only characterize a portion of total utility using observed data, so we assume the unobserved portion to be randomly distributed:

$$u_{i,k} = v_{i,k} + \varepsilon_{i,k}, \quad \varepsilon_{i,k} \xleftarrow{\text{i.i.d.}} \text{Gumbel}\,(0, \gamma) \tag{7}$$

where $v_{i,k}$ is the observable portion of utility for i choosing k, and $\varepsilon_{i,k}$ is the unobservable portion, which we take to be drawn identically and independently (i.i.d.) from a Gumbel distribution with location parameter 0 and dispersion parameter γ.

The observable portion we consider to be a systematic function of independent variables $\mathbf{x}$, comprising cost, income, and possibly other attributes of the decision maker i and of the alternative k. In this case we consider only income of the decision maker, and cost and travel time of the alternative:

$$v_{i,k} = v\left(\mathbf{x}_{i,k}\right) = v\left(y_i - p_k, t_k\right), \tag{8}$$

where y_i is the income of individual i, while p_k and t_k are the price and travel time of alternative k.

From the analyst's perspective, we are given the full matrix of decision makers' observable attributes and alternatives' observable characteristics, $\mathbf{x}$, but not the matrix of unobservable components, $\boldsymbol{\varepsilon}$. Because we assumed the unobserved portion to be i.i.d. Gumbel, we can express the estimated probabilities of choosing each alternative as a function of the observed portions of utility for all alternatives:

$$P_{i,k}\left(\mathbf{v}_i\right) = \frac{e^{\gamma v_{i,k}}}{\sum_{l \in \mathcal{A}} e^{\gamma v_{i,l}}} = \frac{e^{v_{i,k}}}{\sum_{l \in \mathcal{A}} e^{v_{i,l}}} \quad \forall \quad i \in \mathcal{P}, k \in \mathcal{A}, \tag{9}$$

where $P_{i,k}$ is the probability of i choosing k, $\mathbf{v}_i$ is the vector of utilities that i derives from each alternative in the choice set $\mathcal{A}$, and γ is the Gumbel distribution's dispersion parameter, also sometimes called the scale parameter, from (7). Note, however, that the value of γ has no bearing on the specification of the deterministic portion of utility, $\upsilon_{i,k}$; therefore its value is conventionally taken to be one. In other words, the unobserved components are taken to be drawn from a standard Gumbel distribution[1].

It follows from (9) that the choice that an individual could make is stochastic, determined jointly across alternatives:

$$\delta_{i,k}(\mathbf{v}_i) = \begin{cases} 1 & \text{with probability } P_{i,k}, \\ 0 & \text{with probability } 1 - P_{i,k} \end{cases} \quad \forall \quad i \in \mathcal{P}, k \in \mathcal{A}, \tag{10}$$

subject to (6), which ensures that exactly one alternative is chosen.

In analyzing a policy, it is usually necessary to characterize the choices that individuals would make under altered conditions that are distinct from those under which the observed choices $\boldsymbol{\delta}$ were made. Under the altered conditions, $\boldsymbol{\delta}$ is truly unknown, so we can evaluate probabilities using (9). However, this may still not be sufficient for planning purposes, such as when a projected number of bus riders is needed for a proposed new route. To provide an estimate, we can simulate a full set of mode choice decisions. For each individual i, we simply compute the systematic components $\mathbf{v}_i$ and draw the random components $\boldsymbol{\varepsilon}_i$, then identify the alternative with the highest total utility.

5 Case Study

As a microcosm of an urban transportation system, consider a single bridge crossing with two mode options, single-occupant automobile and bus transit, and with no viable alternative routes. Traffic on the bridge operates with a capacity constraint that increases travel time for autos as the traffic volume approaches the roadway's capacity. Buses, however, operate on an exclusive lane at freeflow speeds. Consider in particular the morning commute in one direction, and only work trips.

Under the conditions above, it is conceivable that travelers would be fixed in their choices of *whether* to travel, *where* to travel, *when* to travel, and what *route* to take. This leaves only the choice of *mode* as a flexible choice. While this result is not terribly realistic, it is a useful starting point to develop the methodology here, and more complexity may be added later.

We can imagine that each traveler chooses a mode of travel that maximizes his or her utility, subject to a budget constraint. The conditions that each traveler faces when choosing a mode are the prices and other characteristics of each mode. When considering how different individuals respond to these conditions, we must also consider those individuals' own characteristics, at least with respect to their budgets, i.e. incomes. In the simple case described here, I consider only the monetary cost and travel time of a mode alternative, and only the income of the decision-maker.

5.1 Mode Choice Alternatives

In the case of the bridge, we consider two alternatives, auto and bus, denoted by "***a***" and "***b***", respectively. Suppose that individual i's systematic utilities from the alternatives take a Translog form

[1]The standard Gumbel distribution has the probability density function: $f(x) = e^x e^{-e^x}$. See Ben-Akiva & Lerman (1985) for additional details.

(Christensen et al. 1975) with second-order cost and travel time effects:

$$\mathbf{v}_i = \boldsymbol{\beta}\mathbf{x}_i \quad \forall \quad i \in \mathcal{P}, \text{ or} \tag{11a}$$

$$\begin{aligned} v_{i,k} &= v\left(y_i - p_k, t_k\right) \\ &= \beta_{y1} \ln\left(y_i - p_k\right) + \beta_{y2} \ln\left(y_i - p_k\right)^2 \\ &+ \beta_{t1} \ln\left(t_k\right) + \beta_{t2} \ln\left(t_k\right)^2 \\ &+ \beta_{yt} \ln\left(y_i - p_k\right) \ln\left(t_k\right) \\ &\forall \quad i \in \mathcal{P}, k \in \{\boldsymbol{a}, \boldsymbol{b}\} \end{aligned} \tag{11b}$$

where $\boldsymbol{\beta}$ are parameters selected using Maximum Likelihood Estimation on the observed choices, $\boldsymbol{\delta}$. The Translog utility function is particularly flexible in representing the substitutability between multiple factors (such as time and cost) in the systematic portion of utility for a discrete choice model (Greene 1999).

5.2 Congestion Constraint

I employ a static model of congestion, whereby auto travel times increase as more individuals choose auto. The relationship is a modified "BPR Curve":

$$t_{\boldsymbol{a}}\left(\boldsymbol{\delta}_{\boldsymbol{a}}\right) = t_{\boldsymbol{a}}^{\text{ff}}\left[1 + \theta\left(\frac{V_{\boldsymbol{a}}\left(\boldsymbol{\delta}_{\boldsymbol{a}}\right)}{V_{\boldsymbol{a}}^{\max}}\right)^{\phi}\right] = t_{\boldsymbol{a}}^{\text{ff}}\left[1 + \theta\left(\frac{\sum_{i\in\mathcal{P}} \delta_{i,\boldsymbol{a}}}{V_{\boldsymbol{a}}^{\max}}\right)^{\phi}\right], \tag{12}$$

where $t_{\boldsymbol{a}}^{\text{ff}}$ is the freeflow travel time for autos, $V_{\boldsymbol{a}}\left(\boldsymbol{\delta}_{\boldsymbol{a}}\right) = \sum_{i\in\mathcal{P}} \delta_{i,\boldsymbol{a}}$ is the flow of traffic using auto lanes, $V_{\boldsymbol{a}}^{\max}$ is the maximum traffic flow that can be accommodated, and θ and ϕ are parameters calibrated to observed traffic dynamics on the bridge (Transportation Research Board 1965). The choice process for all individuals and the congestion constraint on the auto lanes must be applied iteratively until the congested auto travel time, $t_{\boldsymbol{a}}$, stabilizes.

5.3 *No-Toll* Conditions

The policy case I am testing is the introduction of a peak-hour toll on the bridge for autos. I first use the base set of parameter values below for the *no-toll* scenario, "$\circ$", below:

$$t_{\boldsymbol{a}}^{\text{ff}} = 12 \text{ minutes}, \tag{13a}$$

$$V_{\boldsymbol{a}}^{\max} = 8000 \text{ vehicles/hour}, \tag{13b}$$

$$\theta = 0.15, \tag{13c}$$

$$\phi = 4, \tag{13d}$$

$$p_{\boldsymbol{a}}^{\circ} = \$0.00, \tag{13e}$$

$$t_{\boldsymbol{b}}^{\circ} = 30 \text{ minutes, and} \tag{13f}$$

$$p_{\boldsymbol{b}}^{\circ} = \$2.00. \tag{13g}$$

where values for θ and ϕ are those used in the Puget Sound Travel Demand Model (Cambridge Systematics Inc. & Urban Analytics Inc. 2003), and the base travel time and maximum traffic volume estimates are taken from Avery et al. (2003).

5.4 *With-Toll* Conditions

For the *with-toll* proposal scenario, "•", we impose a toll of $\tau_{\boldsymbol{a}}^{\bullet} = \$3.00$, which increases the cost for the auto mode:

$$p_{\boldsymbol{a}}^{\bullet} = p_{\boldsymbol{a}}^{\circ} + \tau_{\boldsymbol{a}}^{\bullet} = \$3.00. \tag{14}$$

These collected tolls produce a total revenue of:

$$\mathrm{T}^{\bullet} = \sum_{i \in \mathcal{P}} \tau_{\boldsymbol{a}}^{\bullet} \delta_{i,\boldsymbol{a}}^{\bullet}, \tag{15}$$

where $\mathrm{T}^{\bullet}$ is the total toll revenue in scenario "•". We next subtract the operating costs for the toll system, plus any construction costs that are to be recouped using the toll revenue:

$$\Psi^{\bullet} = \mathrm{T}^{\bullet} - \Upsilon^{\bullet} = \sum_{i \in \mathcal{P}} \tau_{\boldsymbol{a}}^{\bullet} \delta_{i,\boldsymbol{a}}^{\bullet} - \Upsilon^{\bullet}, \tag{16}$$

where $\Psi^{\bullet}$ is the surplus toll revenue after operating and construction costs, $\Upsilon^{\bullet}$.

The remaining toll revenue, $\Psi^{\bullet}$, can then be reallocated to the population in some fashion, or not at all. These refund payments have the effect of increasing each individual's expenditure budget by some non-negative amount $\psi_i^{\bullet}$, such that individual i's budget in the *with-toll* scenario is, effectively, $y_i + \psi_i^{\bullet}$. In allocating toll revenues, I will consider four different scenarios: Scenario $\mathbb{A}$: No Refund; Scenario $\mathbb{B}$: Lump Sum; Scenario $\mathbb{C}$: Transit Subsidy; and Scenario $\mathbb{D}$: Tax Reduction.

Scenario $\mathbb{A}$: No Refund In the first scenario, toll revenues are not returned in any form, but rather "burned":

$$\text{Scenario } \mathbb{A}\text{: } \psi_i^{\bullet} = 0 \quad \forall \quad i \in \mathcal{P}. \tag{17}$$

This somewhat unrealistic scenario serves the purpose of isolating the redistributional effects of the congestion pricing system itself, in isolation from the toll revenue refund scheme. This scenario would actually be the case if there were no surplus toll revenues to be refunded, or $\Psi^{\bullet} = 0$.

Scenario $\mathbb{B}$: Lump Sum In this scenario, revenues are returned to the study population in equal payments:

$$\begin{aligned} \text{Scenario } \mathbb{B}\text{: } \psi_i^{\bullet} &= \frac{1}{N_{\mathcal{P}}} \times \Psi^{\bullet} \\ &= \frac{1}{N_{\mathcal{P}}} \left(\sum_{j \in \mathcal{P}} \tau_{\boldsymbol{a}}^{\bullet} \delta_{j,\boldsymbol{a}}^{\bullet} - \Upsilon^{\bullet} \right) \quad \forall \quad i \in \mathcal{P}, \end{aligned} \tag{18}$$

where $N_{\mathcal{P}}$ is the total number of decision-makers. This scenario essentially simulates a "lump sum" payment to all individuals.

Scenario $\mathbb{C}$: Transit Subsidy The third scenario returns revenues in proportion to transit trips:

$$\begin{aligned} \text{Scenario } \mathbb{C}\text{: } \psi_i^{\bullet} &= \frac{\delta_{i,\boldsymbol{b}}^{\bullet}}{\sum_{j \in \mathcal{P}} \delta_{j,\boldsymbol{b}}^{\bullet}} \times \Psi^{\bullet} \\ &= \frac{\delta_{i,\boldsymbol{b}}^{\bullet}}{\sum_{j \in \mathcal{P}} \delta_{j,\boldsymbol{b}}^{\bullet}} \left(\sum_{j \in \mathcal{P}} \tau_{\boldsymbol{a}}^{\bullet} \delta_{j,\boldsymbol{a}}^{\bullet} - \Upsilon^{\bullet} \right) \quad \forall \quad i \in \mathcal{P}. \end{aligned} \tag{19}$$

Scenario $\mathbb{C}$ is meant to simulate the benefits each individual would receive if the revenues were used to improve transit service, such as by lowering the fare.

Scenario $\mathbb{D}$: Tax Reduction In the final scenario, we return revenues in proportion to income level:

$$\begin{aligned} \text{Scenario } \mathbb{D}\text{: } \psi_i^\bullet &= \frac{y_i}{\sum_{j\in\mathcal{P}} y_j} \times \Psi^\bullet \\ &= \frac{y_i}{\sum_{j\in\mathcal{P}} y_j} \left(\sum_{j\in\mathcal{P}} \tau_{\boldsymbol{a}}^\bullet \delta_{j,\boldsymbol{a}}^\bullet - \Upsilon^\bullet \right) \quad \forall \quad i \in \mathcal{P}. \end{aligned} \tag{20}$$

This scenario approximates an across-the-board tax reduction to all individuals using the generated toll revenues.

In all of the above cases except Scenario $\mathbb{A}$, the inclusion of refund payments will increase the expenditure budgets for at least a significant portion, if not all, of the simulated decision makers. If the utility equation given in (11b) includes more than first-order linear income effects, then this change in *effective* income could be enough to change some individuals' mode choices. That, in turn, can change how many people are paying the toll, which in turn affects the total revenue collected. In Scenario $\mathbb{C}$, mode choice switches can also cause a change in how the toll revenue is allocated to individuals. Hence, some iteration will be required to ensure that the simulated mode choices and the refund payments are consistent. I will discuss the iteration procedure in Section 5.6 below.

5.5 Constancy of the Unobserved Utility

In this case study, I take the assumption that the unobserved portion of utility in both the *no-toll* and *with-toll* scenarios are identical, so:

$$\boldsymbol{\varepsilon}^\circ = \boldsymbol{\varepsilon}^\bullet = \boldsymbol{\varepsilon}. \tag{21}$$

This is a limiting assumption, but a common one taken in discrete choice modeling. The interpretation of this assumption is that the proposed policy change does not change any of the unobservable factors that affect the population's utilities and, hence, choice behaviors. For example, if a commuter tends to drop a child off at school on the way to work, making auto more attractive for unobserved reasons, then this is the case both before and after the toll is introduced, and therefore consistent with (21). Conversely, if the individual values the *reliability* of travel times (as distinct from *shorter* travel times) and benefits from increased reliability after the toll is introduced, then this would go unobserved and be present only in the *with-toll* case, thereby violating the assumption in (21).

5.6 Iteration

As we saw in Sections 5.2 and 5.4, two quantities need to be updated as individuals' mode choice decisions are simulated: a) the congested travel time for autos, $t_{\boldsymbol{a}}$, and b) each individual's budget, $y_i + \psi_i^\bullet$, after accounting for the toll revenue refund payment, $\psi_i^\bullet$. To accomplish this, we first assume initial values as follows:

$$t_{\boldsymbol{a}}^{\bullet,0} = t_{\boldsymbol{a}}^{\text{ff}}, \tag{22}$$

$$\psi_i^{\bullet,0} = \$0.00 \quad \forall \quad i \in \mathcal{P}, \tag{23}$$

and we simulate initial decisions. We then update the congested auto travel times $t_{\boldsymbol{a}}^{\bullet}$ using (12) to adapt travel times in the new iteration to traffic volumes in the previous iteration:

$$t_{\boldsymbol{a}}^{\bullet,n+1} = t_{\boldsymbol{a}}^{\text{ff}} \left[1 + \theta \left(\frac{\sum_{i \in \mathcal{P}} \delta_{i,\boldsymbol{a}}^{\bullet,n}}{V_{\boldsymbol{a}}^{\max}} \right)^{\phi} \right] \quad \forall \quad n > 0, \tag{24}$$

where n indicates the iteration number.

We also update the refund payments using (17) through (20), depending on the refund scheme selected. For example, with the Scenario $\mathbb{C}$ refund scheme, we update the refund payments as follows:

$$\psi_i^{\bullet,n+1} = \frac{\delta_{i,\boldsymbol{b}}^{\bullet,n}}{\sum_{j \in \mathcal{P}} \delta_{j,\boldsymbol{b}}^{\bullet,n}} \left(\sum_{j \in \mathcal{P}} \tau_{\boldsymbol{a}}^{\bullet} \delta_{j,\boldsymbol{a}}^{\bullet,n} - \Upsilon^{\bullet} \right) \quad \forall \quad i \in \mathcal{P}, n > 0. \tag{25}$$

Using the updated congested auto travel times and individual budgets with refunds, we proceed to simulate again the population's choices, and continue to update values and choices cyclicly, until the auto travel times and the revenue payments stabilize.

6 Welfare Analysis

This section is concerned with how to measure welfare levels for each individual, under each potential policy scenario. In this discussion I begin to use an individual's income as a surrogate for well-being or welfare, at least in the base case, and to interpret welfare in other cases to be additive with income, other costs and refunds, and a monetized representation of utility provided by an alternative.

6.1 Compensating and Equivalent Variations for Welfare Analysis

The Hicksian welfare measures "compensating variation" and "equivalent variation" are commonly used in econometric modeling to quantify the effects of a policy change on the welfare of a population (Hicks 1939). These measures use the systematic representations of individuals' utility functions to estimate monetary equivalents of the policy change, with the first measure using the base case as a reference point, and the second using the policy case as a reference point.

First, the compensating variation (CV) is the amount of monetary compensation to individual i that would be required to restore that individual to her original utility level, after prices and qualities have changed due to a policy change that occurs. The compensating variation is implicitly given by:

$$cv_i^{\circ\bullet} = \arg\Big\{ cv : \underbrace{u\left(y_i, \mathbf{p}^{\circ}, \mathbf{t}^{\circ}, \boldsymbol{\varepsilon}_i\right)}_{\substack{\text{uncompensated} \\ \text{base scenario}}} = \underbrace{u\Big(y_i + \psi_i^{\bullet} + cv, \mathbf{p}^{\bullet}, \mathbf{t}^{\bullet}, \boldsymbol{\varepsilon}_i\Big)}_{\substack{\text{compensated} \\ \text{proposal scenario}}} \Big\}. \tag{26}$$

where we be sure to distinguish between an individual's initial budget in the base scenario owing only to income, y_i, and an individual's adjusted budget level in the proposed scenario when toll revenues are refunded, $y_i + \psi_i^{\bullet}$. Another way to interpret the CV is as either the maximum amount of money that an individual would be willing to pay for a beneficial proposal, or as the minimum amount that the individual would accept as compensation for a detrimental proposal.

Second, the equivalent variation (EV) is the amount of monetary compensation that would alter individual i's utility to a new utility level that is equivalent to the utility level that some policy change would effect, where that policy change does not actually occur. The equivalent variation implicitly satisfies the following:

$$ev_i^{\circ\bullet} = \arg\Big\{ev : \underbrace{u\left(y_i + ev, \mathbf{p}^{\circ}, \mathbf{t}^{\circ}, \boldsymbol{\varepsilon}_i\right)}_{\substack{\text{compensated}\\ \text{base scenario}}} = \underbrace{u\left(y_i + \psi_i^{\bullet}, \mathbf{p}^{\bullet}, \mathbf{t}^{\bullet}, \boldsymbol{\varepsilon}_i\right)}_{\substack{\text{uncompensated}\\ \text{proposal scenario}}}\Big\}. \tag{27}$$

The EV can also be interpreted as either the minimum amount of money that an individual would be willing to accept in lieu of a beneficial proposal, or the maximum amount that the individual would be willing to pay to avoid a detrimental proposal.

Note that the CV in (26) and the EV (27) are identical except for a reversal of scenario superscripts, meaning we can say that the equivalent variation of moving from scenario "$\circ$" to scenario "$\bullet$" is identical to the compensating variation of moving from scenario "$\bullet$" to scenario "$\circ$":

$$ev_i^{\circ\bullet} = cv_i^{\bullet\circ}. \tag{28}$$

Note also that the utility expressions in (26) and (27) are unconditional on the choices i makes in each scenario; recall from (4) that the function $u(\cdot)$, given the attributes of all alternatives, finds and chooses the alternative that maximizes utility. Because this determination depends on the unobservable utilities $\boldsymbol{\varepsilon}_i$, which are taken to be random from the analysts point of view, the value of the CV is also stochastic from the analyst's point of view.

6.2 The CV for Discrete Choice Without Income Effects

I proceed for now by discussing only the CV, but the results apply analogously to the EV, as we will see later. In the context of a binomial logit discrete choice situation, the CV has a simple form, provided that one assumes that utility from each alternative is only a first-order linear function of income—what is sometimes referred to as the "no income-effects" model, since the first-order income term has the identical effect on utility for all alternatives, eliminating it as a factor in the resulting choice. Also under this restriction, the marginal utility of money is constant, regardless of income level. A series of works developed the widely used "log-sum" formula for the "no income-effects" model (Domencich & McFadden 1975, Williams 1977, McFadden 1978, Small & Rosen 1981):

$$\mathbb{E}\left[cv_i^{\circ\bullet}\right] = \frac{1}{\beta_y}\left[\ln\sum_{k\in\mathcal{A}} e^{v_{i,k}^{\bullet}} - \ln\sum_{k\in\mathcal{A}} e^{v_{i,k}^{\circ}}\right] \quad \forall \quad i \in \mathcal{P}, \tag{29}$$

where β_y is the marginal utility of money, which is found through the maximum likelihood estimation of the linear-in-income utility equation's coefficients.

The CV above is trivial to compute, making it attractive for use in transportation policy welfare analyses, among them works by Rodier & Johnston (1997), and Kalmanje & Kockelman (2004), as well as in a wide variety of applications outside transportation policy. Some past studies, such as by Niemeier (1997), do include a non-linear income term, but it acts only as an indicator of taste, rather than as an explicit representation of a decision-maker's budget. As such, while the utility function can produce some sensitivity of decision behavior to income, we still have the assumption of constant marginal utility of income, which is critical when computing an individual's CV or EV for a policy change.

Jara-Diaz & Videla (1989) have shown, however, that the marginal utility of income can be distinctly different, depending on income group, especially when examining a wide range of income levels. Significantly, they found that a person in a lower income group tends to derive significantly greater utility from each additional dollar than someone from a higher income group, i.e. there is a diminishing marginal utility to money. In a conventional aggregate welfare analysis, the consequence of ignoring income effects is that the constant marginal utility would cause costs and benefits accruing to lower income individuals to be undervalued relative to the costs and benefits to higher income individuals. While this is cause for concern in any study of welfare effects, it is especially concerning in a welfare study such as this one that focuses on the equitable aspects of a welfare distribution, and on how that distribution changes with a proposed policy. For this reason, the "logsum" formula in (29) is a somewhat undesirable formula for estimating CV and EV.

6.3 The CV for Discrete Choice With Income Effects

To estimate the welfare effect of a policy change, I adapt the compensating variations measure described in Karlström (2001) and Karlström & Morey (2001), which allows for non-linear income effects in Generalized Extreme Value Random Utility Models, with the only modification being that I include the term $\psi_i^\bullet$, representing a toll revenue refund payment that is only present in the *with-toll* case.

In Karlström's formulation, we evaluate the compensating variation in terms of the expected *total* expenditure level necessary to maintain an individual's initial utility, such that:

$$\mathbb{E}\left[m_i^\bullet\right] = y_i + \psi_i^\bullet + \mathbb{E}\left[cv_i^{\circ\bullet}\right], \tag{30}$$

where $m_i^\bullet$ is the total compensated expenditure level for individual i in scenario "$\bullet$" that, when combined with the refund payment $\psi_i^\bullet$, restores i's original level of utility from scenario "$\circ$", in which i had an expenditure level of y_i.

Karlström demonstrates that the expected compensated expenditure level is given by:

$$\begin{aligned} \mathbb{E}\left[m_i^\bullet\right] &= \sum_{k\in\mathcal{A}} \left\{ \mu_{i,kk}^\bullet P_{i,k}^*\left(\mu_{i,kk}^\bullet\right) - \int_{\underline{\mu_i^\bullet}}^{\mu_{i,kk}^\bullet} \mu \frac{\partial P_{i,k}^*(\mu)}{\partial \mu} d\mu \right\} \\ &= \sum_{k\in\mathcal{A}} \left\{ \mu_{i,kk}^\bullet P_{i,k}^*\left(\mu_{i,kk}^\bullet\right) - \int_{\underline{\mu_i^\bullet}}^{\mu_{i,kk}^\bullet} \mu \, dP_{i,k}^*(\mu) \right\} \quad \forall \quad i \in \mathcal{P}, \end{aligned} \tag{31}$$

where $\mu_{i,kk}^\bullet$ is the expenditure level that would maintain individual i's original utility, given that i chose alternative k in *both* scenarios, "$\circ$" and "$\bullet$", such that the following is satisfied:

$$v\left(y_i - p_k^\circ, t_k^\circ \mid \delta_{i,k}^\circ = 1\right) = v\left(\mu_{i,kk}^\bullet + \psi_i^\bullet - p_k^\bullet, t_k^\bullet \mid \delta_{i,k}^\bullet = 1\right). \tag{32}$$

Meanwhile, $\underline{\mu_i^\bullet}$ in (31) is the minimum of these expenditure levels across all alternatives:

$$\underline{\mu_i^\bullet} = \min_{k\in\mathcal{A}} \mu_{i,kk}^\bullet \quad \forall \quad k \in \mathcal{A}. \tag{33}$$

Also in (31), the constructed choice probability $P_{i,k}^*(\mu)$ for i choosing k in both scenarios, given a competing expenditure level μ, is given by:

$$P_{i,k}^*\left(\mu \mid y_i, \psi_i^\bullet, \mathbf{p}^\circ, \mathbf{t}^\circ, \mathbf{p}^\bullet, \mathbf{t}^\bullet\right) = \frac{e^{v\left(y_i - p_k^\circ, t_k^\circ\right)}}{e^{v\left(y_i - p_k^\circ, t_k^\circ\right)} + \sum_{l\neq k} e^{\max\left[v\left(y_i - p_l^\circ, t_l^\circ\right), v\left(\mu + \psi_i^\bullet - p_l^\bullet, t_l^\bullet\right)\right]}}, \tag{34}$$

where the systematic utility v is given by (11b).

6.4 The Income Effects CV for the Binomial Case

In the specific context here, we have a binomial mode decision between auto and bus, and furthermore we know that the only changes between scenarios are an increase in cost for auto and a decrease in travel times for auto (bus costs and travel times remain the same); and we also have a toll revenue refund payment that is added to each individual's budget.

Under these constraints, the expected compensated expenditure level for individual i, unconditional of mode choice before and after, is:

$$
\begin{aligned}
\mathbb{E}\left[m_i^\bullet\right] &= c_{\boldsymbol{bb}} + c_{\boldsymbol{aa}} + c_{\boldsymbol{ab}} + c_{\boldsymbol{ba}} \\
&= \mu_{i,\boldsymbol{aa}}^\bullet P_{i,\boldsymbol{a}}^* \left(\mu_{i,\boldsymbol{aa}}^\bullet\right) + \left(y_i - \psi_i^\bullet\right) P_{i,\boldsymbol{b}}^* \left(y_i - \psi_i^\bullet\right) \\
&\quad - \begin{cases} \int_{y_i-\psi_i^\bullet}^{\mu_{i,\boldsymbol{aa}}^\bullet} \mu \, dP_{i,\boldsymbol{a}}^* (\mu) & \text{if } \mu_{i,\boldsymbol{aa}}^\bullet > y_i - \psi_i^\bullet, \\ \int_{\mu_{i,\boldsymbol{aa}}^\bullet}^{y_i-\psi_i^\bullet} \mu \, dP_{i,\boldsymbol{b}}^* (\mu) & \text{if } \mu_{i,\boldsymbol{aa}}^\bullet < y_i - \psi_i^\bullet, \\ 0 & \text{otw.} \end{cases}
\end{aligned} \tag{35}
$$

where $\mu_{i,\boldsymbol{aa}}^\bullet$ is given by:

$$
\begin{aligned}
\mathbb{E}\left[m_{i,\boldsymbol{aa}}^\bullet\right] = \mu_{i,\boldsymbol{aa}}^\bullet &= \mu\left(y_i - p_{\boldsymbol{a}}^\circ, t_{\boldsymbol{a}}^\circ; \psi_i^\bullet - p_{\boldsymbol{a}}^\bullet, t_{\boldsymbol{a}}^\bullet\right) \\
&= \arg\left[\mu : v\left(y_i - p_{\boldsymbol{a}}^\circ, t_{\boldsymbol{a}}^\circ\right) = v\left(\mu + \psi_i^\bullet - p_{\boldsymbol{a}}^\bullet, t_{\boldsymbol{a}}^\bullet\right)\right],
\end{aligned} \tag{36}
$$

and the probabilities $P_{i,\boldsymbol{a}}^*$ and $P_{i,\boldsymbol{b}}^*$ are given by:

$$
P_{i,k}^* (\mu) = \frac{e^{v\left(y_i^\circ - p_k^\circ, t_k^\circ\right)}}{e^{v\left(y_i^\circ - p_k^\circ, t_k^\circ\right)} + e^{\max\left[v\left(y_i^\circ - p_l^\circ, t_l^\circ\right), v\left(\mu + \psi_i^\bullet - p_l^\bullet, t_l^\bullet\right)\right]}} \quad \forall \quad k, l \in \{\boldsymbol{a}, \boldsymbol{b}\}, k \neq l. \tag{37}
$$

Note that only one of the integral terms in (70) will remain, depending on whether $\mu_{i,\boldsymbol{aa}}^\bullet$ is greater or less than the original expenditure level (i.e. income) minus the refund payment, $y_i - \psi_i^\bullet$; this determination, in turn, depends on the specification of the utility function and the values its coefficients, as well as on the amount of the refund. In the general form given by (31), this selection of integral terms is accomplished by identifying the minimum compensated expenditure level by (33) and using it as the lower bound of the integrals.

For a detailed explanation of the Compensating Variations measure with income effects for the binomial case discussed here, please see the Appendix.

6.5 The Income Effects EV for the Binomial Case

The equivalent variation can be evaluated in a similar manner to the compensating variation. We evaluate the equivalent variation in terms of the expected total expenditure level necessary in the base scenario to meet an individual's utility under the uncompensated proposed policy conditions, such that:

$$
\mathbb{E}\left[m_i^\circ\right] = y_i^\bullet + \mathbb{E}\left[ev_i^{\circ\bullet}\right], \tag{38}
$$

where scenario '•' is the proposed policy conditions with an uncompensated expenditure level, but with a toll refund payment, giving a total budget $y_i + \psi_i^\bullet$, and where scenario '○' is the base policy conditions with a compensated expenditure level without toll refund payment, $y_i + ev_i^{\circ\bullet}$.

Noting the equivalency in (28), we can proceed analogously to the compensating variations case by simply reversing the policy conditions but using the expenditure level $y_i^\bullet$ as the reference point (now, in the proposed policy scenario). We then express the expected expenditure level in the compensated base case, from (38), as:

$$\mathbb{E}\left[m_i^\circ\right] = \mu_{i,\boldsymbol{aa}}^\circ P_{i,\boldsymbol{a}}^*\left(\mu_{i,\boldsymbol{aa}}^\circ\right) + \left(y_i + \psi_i^\bullet\right) P_{i,\boldsymbol{b}}^*\left(y_i + \psi_i^\bullet\right) \\ - \begin{cases} \int_{y_i+\psi_i^\bullet}^{\mu_{i,\boldsymbol{aa}}^\circ} \mu \, dP_{i,\boldsymbol{a}}^*(\mu) & \text{if } \mu_{i,\boldsymbol{aa}}^\circ > y_i + \psi_i^\bullet, \\ \int_{\mu_{i,\boldsymbol{aa}}^\circ}^{y_i+\psi_i^\bullet} \mu \, dP_{i,\boldsymbol{b}}^*(\mu) & \text{if } \mu_{i,\boldsymbol{aa}}^\circ < y_i + \psi_i^\bullet, \\ 0 & \text{otw,} \end{cases} \tag{39}$$

where $\mu_{i,\boldsymbol{aa}}^\circ$ is implicitly given by:

$$\mu_{i,\boldsymbol{aa}}^\circ = \mu\left(y_i + \psi_i^\bullet - p_{\boldsymbol{a}}^\bullet, t_{\boldsymbol{a}}^\bullet; -p_{\boldsymbol{a}}^\circ, t_{\boldsymbol{a}}^\circ\right) \\ = \arg\left[\mu : v\left(y_i + \psi_i^\bullet - p_{\boldsymbol{a}}^\bullet, t_{\boldsymbol{a}}^\bullet\right) = v\left(\mu - p_{\boldsymbol{a}}^\circ, t_{\boldsymbol{a}}^\circ\right)\right], \tag{40}$$

and the probabilities $P_{i,\boldsymbol{a}}^*$ and $P_{i,\boldsymbol{b}}^*$ are both evaluated using:

$$P_{i,k}^*(\mu) = \frac{e^{v\left(y_i+\psi_i^\bullet-p_k^\bullet, t_k^\bullet\right)}}{e^{v\left(y_i+\psi_i^\bullet-p_k^\bullet, t_k^\bullet\right)} + e^{\max\left[v\left(y_i+\psi_i^\bullet-p_l^\bullet, t_l^\bullet\right), v\left(\mu-p_l^\circ, t_l^\circ\right)\right]}} \\ \forall \quad k, l \in \{\boldsymbol{a}, \boldsymbol{b}\}; k \neq l. \tag{41}$$

6.6 Total Welfare Estimates

We now have methods for estimating the compensating and equivalent variations for the binary mode choice model I presented in Section 5, with a non-linear income effect. Each of these measures is an indicator of the *change* in consumer welfare between one scenario and another. However, my interest is not only in welfare change, but in the change in how *total* welfare is distributed from one scenario to the next. As an indicator of an individual's absolute welfare in the base case, I use the individual's income, y_i.

Then, to measure the increment, I use the equivalent variation. To understand why I choose the EV instead of the CV, consider that in my conceptualization of a policy's effects, each individual experiences a tangible *change* in absolute welfare. The EV, by its definition, seeks the monetary payment that would cause a *change* in welfare that is equivalent to that which would be caused by the policy change. The CV, on the other hand, is concerned not with a change in welfare, but with the *negation* of a change to leave welfare as it was. Such a measure is therefore uninformative for the purpose of assessing a change in absolute welfare.

The estimates of total welfare in each scenario are operationalized as follows:

$$\left.\begin{aligned} w_i^\circ &= y_i \\ w_i^\bullet &= y_i + \mathbb{E}\left[ev_i^{\circ\bullet}\right] \end{aligned}\right\} \quad \forall \quad i \in \mathcal{P}. \tag{42}$$

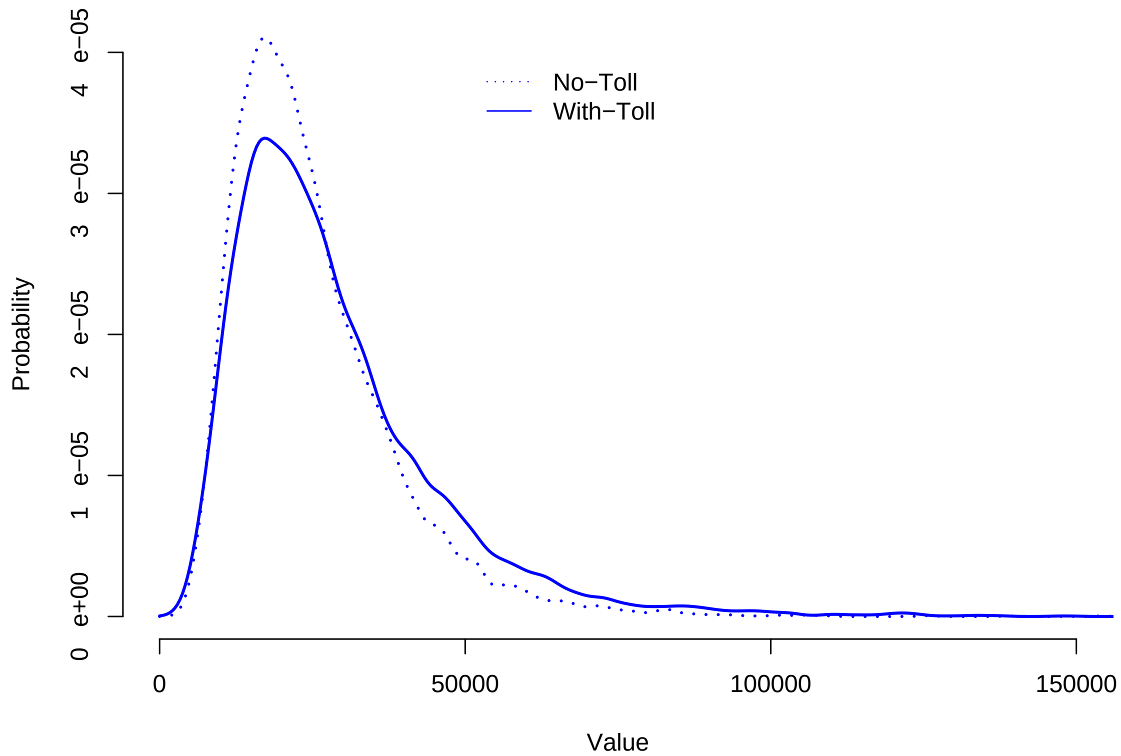
Collectively over all individuals in $\mathcal{P}$, these constitute the welfare distribution vectors $\mathbf{w}^\circ$ and $\mathbf{w}^\bullet$ for the *no-toll* and *with-toll* scenarios, respectively.

7 Distributional Analysis

This research design uses a variety of methods to analyze the welfare distributions that emerge from the above welfare analysis. Several of these methods deserve some initial introduction before the formal hypothesis tests are stated.

To demonstrate these methods, I will refer to a synthesized example dataset, where we have a population of 1,000, and each individual has some welfare level in each of two scenarios, the *no-toll* and *with-toll* scenarios, giving us $\mathbf{w}^{\circ}$ and $\mathbf{w}^{\bullet}$. These data were generated from a log-normal distribution, where the *no-toll* and *with-toll* scenarios have log-means of 10 and 10.1, and log-standard-deviations of 0.5 and 0.55, respectively. Several of the tools below require us to treat each welfare distribution vector $\mathbf{w}$ as a set of draws from a random variable W with some unknown, smooth probability distribution, which can be described in terms of its cumulative density function (CDF), F_W, and its probability density function (PDF), f_W. For simplicity, from here forward I omit the subscript "W" in F and f. The PDFs for the synthesized example dataset in each scenario are shown in Figure 1.

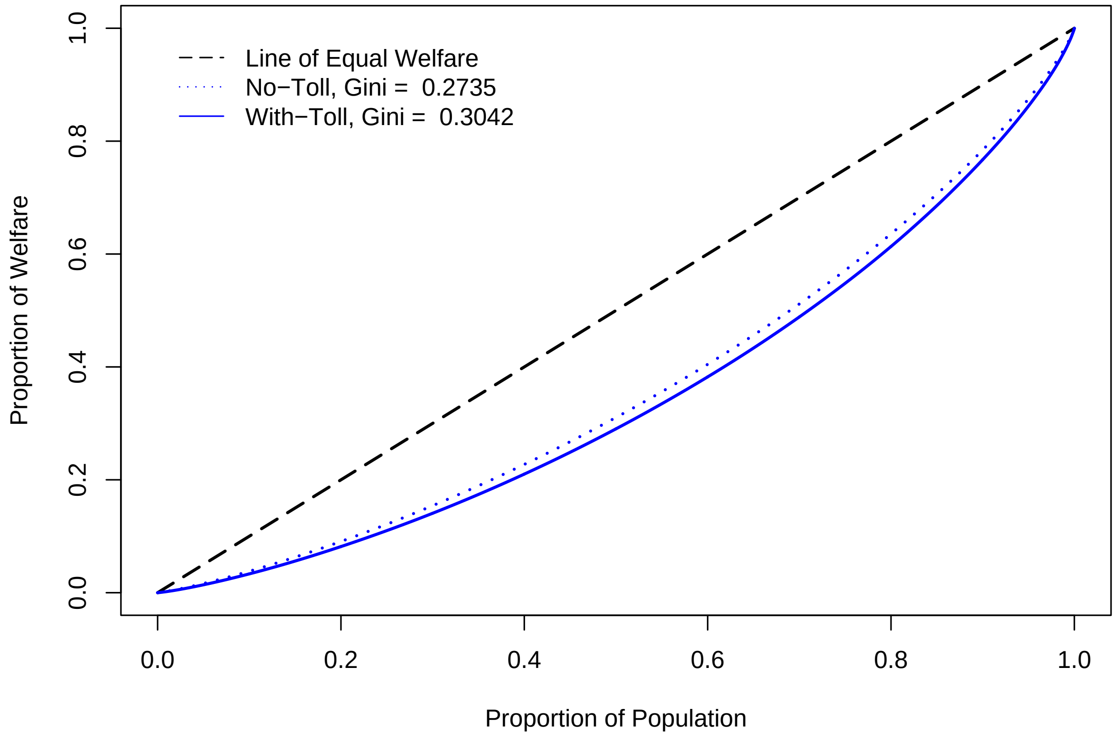
Figure 1: Example Data PDFs



7.1 Gini Coefficient

The *Gini Coefficient* (Gini 1912) is very common in empirical equity literature. It is perhaps most easily understood graphically using the Lorenz Curve (Lorenz 1905). To construct a Lorenz Curve, we order our welfare estimates from lowest to highest, then plot the cumulative proportion of total welfare against the cumulative proportion of total population. This results in a convex curve bounded at its low end by [0,0] and at its high end by [1,1], as shown by the example in Figure 2. Usually, a Lorenz Curve is superimposed over a straight diagonal line from [0,0] to [1,1], which represents what the Lorenz Curve would look like if everyone had exactly the same welfare.

Figure 2: Example Lorenz Curve



The Gini Coefficient, then, is twice the ratio between two areas: the numerator is the area bounded by the Lorenz Curve and the diagonal, and the denominator is the total area below the diagonal. If the welfare distributions were truly equal for all individuals, then the Gini Coefficient would equal zero, and if all welfare were concentrated with one individual, then it would equal one.

The Gini Coefficient can also be computed without the aid of a Lorenz Curve, using the following formula (Dixon et al. 1987):

$$\mathcal{G}(\mathbf{w}) = \frac{1}{N_{\mathcal{P}}^2 \overline{w}_{\mathcal{P}}} \sum_{i \in \mathcal{P}} \sum_{j \in \mathcal{P}} \left| w_i - w_j \right|, \tag{43}$$

where $N_{\mathcal{P}}$ is the number of individuals in population $\mathcal{P}$, and $\overline{w}_{\mathcal{P}} = \sum_{i \in \mathcal{P}} w_i / N_{\mathcal{P}}$ is the mean welfare level across population $\mathcal{P}$.

7.2 The Relative Distribution

A tool that will help with several of the hypotheses is the *relative distribution* (RD), which is described in detail by Handcock & Morris (1999). When comparing our two policy scenarios' welfare distributions $\mathbf{w}^{\circ}$ and $\mathbf{w}^{\bullet}$, the RD is defined by its random variable, which is essentially a transformation of the *with-toll* distribution's random variable, $W^{\bullet}$, using the CDF of the *no-toll* distribution, F° as the transforming function:

$$R^{\bullet\circ} = F^{\circ}(W^{\bullet}), \tag{44}$$

where $R^{\bullet\circ}$ is the random variable for the relative distribution between scenarios "•" and "○".

We are most often interested in the PDF of the RD, and heretofore when I refer to the RD as a graphical tool, I refer specifically to its PDF. The PDF is also more intuitive to understand: a relative distribution's PDF plots the ratio between the two distributions' probability densities against the quantiles of the *no-toll* distribution:

$$g^{\bullet\circ}(r) = \frac{f^{\bullet}(Q^{\circ}(r))}{f^{\circ}(Q^{\circ}(r))} \quad \forall \quad 0 \le r \le 1, \tag{45}$$

where $g^{\bullet\circ}(r)$ is the PDF of the relative distribution $R^{\bullet\circ}$, $Q^{\circ}(r)$ is the quantile function[2] of the *no-toll* distribution, and r is some quantile level in the range 0 to 1.

As a ratio between two non-negative quantities, the RD can take on y-values ranging from 0 to $+\infty$, and values of 1 indicate that the two distributions have the same probability density at that welfare quantile level. Hence, a uniform density of the RD of 1, across its entire support $[0, 1]$, would indicate that the two distributions are identical. Note that since the RD is actually a probability density function, the area underneath the curve will always equal one. Figure 3 shows an example of an RD, produced from the example data. The shape of the curve indicates that the *with-toll* distribution is sparser in the lower welfare ranges, and denser in the upper welfare ranges, than the *no-toll* distribution, from which we can conclude that in general, individuals have welfares of values in the *with-toll* scenario than the *no-toll* scenario.

7.3 Kullback-Leibler Divergence

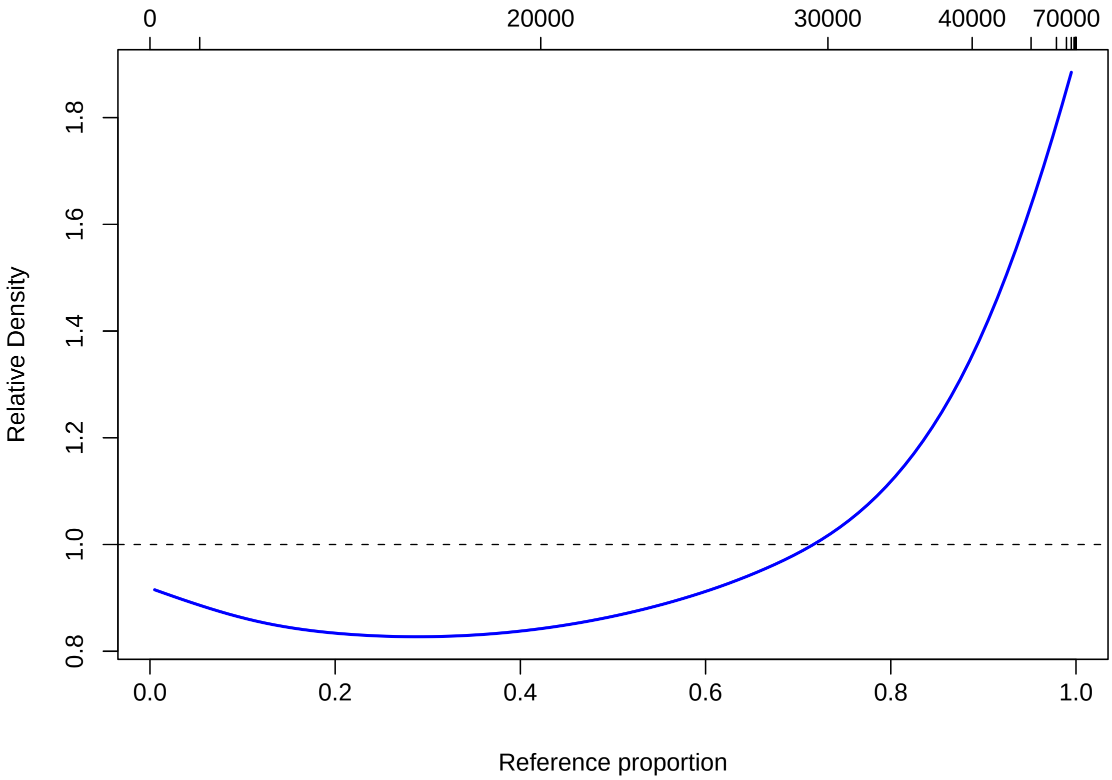
The *Kullback-Leibler Divergence* (KL) is an information theoretic measure of whether two distributions are equivalent (Kullback & Leibler 1951), and it can be expressed either in terms of the two distributions' PDFs or of the PDF of their relative distribution:

$$\begin{aligned} \mathrm{KL}(f^{\bullet}; f^{\circ}) &= \int_{-\infty}^{\infty} \log\left(\frac{f^{\bullet}(w)}{f^{\circ}(w)}\right) dF^{\circ}(w) \\ &= \mathrm{Entropy}(g^{\bullet\circ}) = \int_{0}^{1} \log(g^{\bullet\circ}(r))\, g^{\bullet\circ}(r)\, dr. \end{aligned} \tag{46}$$

Hence, the KL is simply the entropy of the relative distribution. This can be interpreted in several ways. Most simply, it measures the amount of information contained in the relative distribution, $g(r)$; hence, if the two distributions being compared were identical, $g(r)$ would be a uniform distribution, containing no information, and the KL would be zero. Relative distributions that deviate from the uniform line will have greater values for the KL, and those values will be weighted more highly if the deviations occur at the margins of the distributions, rather than near its central tendency.

[2] Recall that the quantile function is the inverse function of the CDF: $Q(r) = F^{-1}(w)$.

Figure 3: Example Relative Distribution



7.4 Mean Relative Polarization

The *Mean Relative Polarization Index*, MRP, measures differences in the shape of the distribution, apart from changes in location. To do this, the measure uses a transformation of the *with-toll* distribution, F°_{Λ}, in which the distribution is location-shifted so that its mean matches that of the *with-toll* distribution, $F^{\bullet}$. The basis for this measure becomes the relative distribution between these two mean-matched distributions:

$$\mathrm{MRP}(F^{\bullet}; F^{\circ}) = 4 \int_0^1 \left| r - \tfrac{1}{2} \right| g^{\bullet\circ}_{\Lambda}(r) dr - 1, \tag{47}$$

where $g^{\bullet\circ}_{\Lambda}(r)$ is the PDF of the mean-adjusted relative distribution $R^{\bullet\circ}_{\Lambda} = F^{\circ}_{\Lambda}(W^{\bullet}) = F^{\circ}(W^{\bullet} - \rho))$, with ρ being the difference in medians between the *no-toll* and *with-toll* distributions.

The MRP, then, measures the height of the mean-matched relative distribution curve, weighted by how far each part of the curve is from the one-half point (hence the term in the absolute-value brackets). When $\mathrm{MRP}(F^{\bullet}; F^{\circ})$ is positive, the *with-toll* distribution's density is located further to its edges than the *no-toll* distribution's density, indicating a *polarizing* effect of the policy. When it is negative, the opposite is the case, i.e. the policy is *de*-polarizing. When the value is zero, the two distributions are identical in shape (although not necessarily in location).

8 Hypothesis Tests & Measures

Recall the hypotheses from Section 3.1. After computing the welfare distributions $\mathbf{w}^\circ$ and $\mathbf{w}^\bullet$, we can now use the methods described in Section 7 to analyze the differences between the distributions with respect to each of these questions, choosing in each case an appropriate test.

Hypothesis 1. Does the policy make a substantial difference? By this, I do not mean whether the policy changes the *central tendency* of the distribution, but whether the distribution can be said to contain different information, whether that information is in its central tendency (e.g. a mean or median) or in the thickness of its tails. To examine this quantitatively, we use an entropy-based Kullback-Leibler Divergence measure. If the two contain indifferentiable information, then this will be zero:

$$\mathrm{H}_1^0 : \mathrm{KL}\left(f^\bullet, f^\circ\right) \equiv \mathrm{KL}\left(g^{\bullet\circ}\right) = 0, \tag{48}$$

where $f^\bullet$ and f° are the Probability Density Functions for the welfare distributions $\mathbf{w}^\bullet$ and $\mathbf{w}^\circ$, respectively, and $g^{\bullet\circ}$ is the Probability Density Function for the relative distribution between them.

Hypothesis 2. Does the policy make an overall improvement? The null hypothesis here can be stated in terms of the sum of all individual welfares in the represented population:

$$\mathrm{H}_2^0 : \Delta w_{\mathcal{P}}^{\bullet\circ} = w_{\mathcal{P}}^\bullet - w_{\mathcal{P}}^\circ = \sum_{i\in\mathcal{P}} w_i^\bullet - \sum_{i\in\mathcal{P}} w_i^\circ = 0, \tag{49}$$

where $w_{\mathcal{P}}$ is the summed welfare for all individuals $i \in \mathcal{P}$.

Hypothesis 3. Does the policy reduce (or increase) the number of individuals under the poverty line? Given an *a priori* poverty line, which denotes a welfare level minimum that we desire all individuals to be at or above, we specify a null hypothesis as:

$$\mathrm{H}_3^0 : \sum_{i\in\mathcal{P}} \mathcal{I}\left[w_i^\bullet < \underline{w}\right] - \sum_{i\in\mathcal{P}} \mathcal{I}\left[w_i^\circ < \underline{w}\right] = 0, \tag{50}$$

where $\underline{w}$ is the specified poverty line.

Hypothesis 4. Is the policy regressive (or progressive)? We operationalize the null hypothesis on "regressivity" using a Gini Coefficient. If the two scenarios are equally regressive, then the coefficients will be equal:

$$\mathrm{H}_4^0 : \mathcal{G}\left(\mathbf{w}^\bullet\right) = \mathcal{G}\left(\mathbf{w}^\circ\right). \tag{51}$$

Hypothesis 5. Does the policy polarize (or depolarize) the spread of welfare levels? For this we use an index of mean-adjusted polarization to determine whether the policy causes a relative increase or decrease in probability density near the extremes of welfare levels. If there is neither polarization nor depolarization, then this will be zero:

$$\mathrm{H}_5^0 : \mathrm{MRP}(F^\bullet; F^\circ) \equiv \mathrm{MRP}(G^{\bullet,\circ}) = 0. \tag{52}$$

Hypothesis 6. Does the policy have different effects on distinct welfare groups? To approach this question, we must first identify the quantile ranges. Here we arbitrarily identify five equally-sized ranges, I through V, using the *no-toll* scenario's quantile function:

$$\begin{aligned} w_{\mathrm{I}}^{\min} &= Q^{\circ}(0), w_{\mathrm{I}}^{\max} = Q^{\circ}(0.2) \\ w_{\mathrm{II}}^{\min} &= Q^{\circ}(0.2), w_{\mathrm{II}}^{\max} = Q^{\circ}(0.4) \\ w_{\mathrm{III}}^{\min} &= Q^{\circ}(0.4), w_{\mathrm{III}}^{\max} = Q^{\circ}(0.6) \\ w_{\mathrm{IV}}^{\min} &= Q^{\circ}(0.6), w_{\mathrm{IV}}^{\max} = Q^{\circ}(0.8) \\ w_{\mathrm{V}}^{\min} &= Q^{\circ}(0.8), w_{\mathrm{V}}^{\max} = Q^{\circ}(1) \end{aligned}$$

We then compute the differences in within-group aggregate sums:

$$\Delta w_{\mathcal{B}}^{\bullet\circ} = w_{\mathcal{B}}^{\bullet} - w_{\mathcal{B}}^{\circ} = \sum_{i\in\mathcal{B}} w_i^{\bullet} - \sum_{i\in\mathcal{B}} w_i^{\circ}, \quad \forall \quad \mathcal{B} \in \{\mathrm{I}, \mathrm{II}, \mathrm{III}, \mathrm{IV}, \mathrm{V}\}. \tag{53}$$

Finally, we compare these within-group differences, with the null hypothesis being that they are all equal:

$$\mathrm{H}_6^0 : \Delta w_{\mathrm{I}}^{\bullet\circ} = \Delta w_{\mathrm{II}}^{\bullet\circ} = \Delta w_{\mathrm{III}}^{\bullet\circ} = \Delta w_{\mathrm{IV}}^{\bullet\circ} = \Delta w_{\mathrm{V}}^{\bullet\circ}. \tag{54}$$

Hypothesis 7. If the policy has heterogenous effects on the welfare distribution, then which parts gain or lose? As in the previous hypothesis, here we are interested in the specific effects of the policy at various locations along the welfare spectrum, but in this case we ask the question without identifying any *a priori* categories, such as the quantiles used above. Instead, we estimate a relative distribution (RD), $R^{\bullet\circ}$, between the two scenarios, using a kernel density estimator to generate a smooth function representing the RD's probability density, $g^{\bullet\circ}$. Recall from Section 7.2 that when the two distributions being compared are identical, the RD's PDF will be equal to one over its entire support, $[0, 1]$. This, then, becomes our null hypothesis:

$$\mathrm{H}_7^0 : g^{\bullet\circ}(r) = 1 \quad \forall \quad 0 < r < 1. \tag{55}$$

If we reject the null hypothesis H_7^0, then we can do more than simply accept the alternate hypothesis; we can also identify which ranges of (initial) welfare-level ranges are gaining, and which are losing, from the policy change, simply by observing which portions of the RD lie above (for an improvement) or below (for a detriment) the horizontal "one" line.

Hypothesis 8. What is the balance between those with a net gain and those with a net loss? Finally, we address the issue of "popularity" by examining the number of winners versus losers (here, I define those whose welfares are unchanged as "winners"). In other words, if the proposed policy were put to a referendum, and each individual voted based on a rational assessment of his or her own welfare only, then would the referendum pass? As a null hypothesis, we take the case that an equal number of individuals gain and lose, so it is unclear whether a referendum on the policy would pass or fail. Our null hypothesis can then be stated:

$$\begin{aligned} &\mathrm{H}_8^0 : N_{\ominus}^{\bullet\circ} = N_{\oplus}^{\bullet\circ} \\ \Rightarrow &\sum_{i\in\mathcal{P}} \mathcal{I}\left[w_i^{\bullet} < w_i^{\circ}\right] = \sum_{i\in\mathcal{P}} \mathcal{I}\left[w_i^{\bullet} \geq w_i^{\circ}\right], \end{aligned} \tag{56}$$

where $N_{\ominus}^{\bullet\circ}$ and $N_{\oplus}^{\bullet\circ}$ are the number of losers and winners due to the change from scenario "∘" to scenario "•", and $\mathcal{I}[\cdot]$ is the indicator function.

We can simplify (56), knowing that the sum of the numbers of winners and losers will equal the total population:

$$\mathrm{H}_8^0 : N_{\ominus}^{\bullet\circ} = \frac{N_{\mathcal{P}}}{2}. \tag{57}$$

If H_8^0 is rejected, then if $N_{\ominus}^{\bullet\circ} > \frac{N_{\mathcal{P}}}{2}$, then we can say that a referendum on the policy would be likely to fail, while with $N_{\ominus}^{\bullet\circ} < \frac{N_{\mathcal{P}}}{2}$, it would be likely to pass.

9 Concluding Remarks

The conceptual framework and research design presented above demonstrates how the distributional effects of a transportation policy can be assessed in a manner that reflects a variety of policy goals, while simultaneously avoiding several of the pitfalls of prior approaches. By using a random utility modeling approach, we allow the unobserved portion of the utility estimates for individuals who change mode between auto and bus to vary across a random distribution. By using a transcendental logarithmic function for the observed utility, we can accommodate flexible substitution patterns between the two major commodities, time and money. The application of a compensating variations measure that incorporates income effects allows the marginal utility of money to vary by income level, and we can avoid understating the effects of a policy on lower-income individuals. Finally, by applying a variety of distributional analysis tools, we can interpret changes in the welfare distributions in ways that directly inform a variety of specific policy questions.

At the same time, this approach is simplistic in several ways that have been improved upon in other studies, and hence the interpretation of any results will be limited. For example, the behavioral modeling framework presented here includes only a single choice dimension, mode-to-work, where only two options are available, auto and bus. In reality, daily travel behavior is a high-dimensioned set of interrelated choices among activities, destinations, routes, modes, and opportunities for coordination with other travelers. This probably means that the results of this study will overstate any real effects of a tested policy, since real travelers will be able to absorb the effect, positive or negative, in a much wider variety of ways.

Second, it is important to note that the congestion model described here is a static one, where there is a monotonic relationship between a roadway's traffic volume and its operating speed. However, observed data on highway operations demonstrate that congestion effects are much more dynamic, exhibiting a cyclical relationship between volume and speed. A more realistic approach would be to apply the bottleneck model from Vickrey (1969), much as Arnott et al. (1993, 1994, 1998) have done. However, the computational expense of the bottleneck model, combined with the iterative processes in the approach presented here, may prevent an integrated approach from being feasible at today's processing speeds.

A final limitation of this approach is that it does not provide inference tests that support the assignment of confidence levels to the results. However, ongoing efforts are underway to provide such an inference testing, using Bayesian methods to estimate the behavioral choice model, and using Monte Carlo simulation to produce distributions of individual-level welfare estimates in each scenario.

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Appendix: The Compensating Variation with Income Effects

To illustrate how we arrive at the compensating variation in (31), let us consider the binomial case of auto versus bus, and furthermore that the only changes between scenarios are an increase in cost for auto and a decrease in travel times for auto; bus costs and travel times remain the same. The following is in large part a synthesis of the explanations given in Karlström (2001) and Karlström & Morey (2001), tailored for the specific application here. In the compensating variations context, we are really considering the following two scenarios:

1. Scenario "∘", with base policy conditions and an uncompensated expenditure level with no toll revenue refund, y_i, and
2. Scenario "•", with proposed policy conditions and a *compensated* expenditure level *with* a toll revenue refund, $y_i + \psi_i^\bullet + cv_i^{\circ\bullet}$.

In this scenario universe, we can imagine four different possibilities for what an individual decision-maker i chooses:

1. Chooses bus both before and after the policy change and compensation (***bb***),
2. Chooses auto both before and after the policy change and compensation (***aa***),
3. Chooses auto before the policy change, but switches to bus afterward (***ab***), and
4. Chooses bus before the policy change, but switches to auto afterward (***ba***).

The expected value of the actual compensation level can be stated as a composite of four components, one for each of the four above possibilities:

$$\begin{aligned} \mathbb{E}\left[m_i^\bullet\right] &= c_{\boldsymbol{bb}} + c_{\boldsymbol{aa}} + c_{\boldsymbol{ab}} + c_{\boldsymbol{ba}} \\ &= \mathbb{E}\left[m_{i,\boldsymbol{bb}}^\bullet\right] \Pr\left[\delta_{i,\boldsymbol{b}}^\circ = \delta_{i,\boldsymbol{b}}^\bullet = 1\right] + \mathbb{E}\left[m_{i,\boldsymbol{aa}}^\bullet\right] \Pr\left[\delta_{i,\boldsymbol{a}}^\circ = \delta_{i,\boldsymbol{a}}^\bullet = 1\right] \\ &\quad + \mathbb{E}\left[m_{i,\boldsymbol{ab}}^\bullet\right] \Pr\left[\delta_{i,\boldsymbol{a}}^\circ = \delta_{i,\boldsymbol{b}}^\bullet = 1\right] + \mathbb{E}\left[m_{i,\boldsymbol{ba}}^\bullet\right] \Pr\left[\delta_{i,\boldsymbol{b}}^\circ = \delta_{i,\boldsymbol{a}}^\bullet = 1\right], \end{aligned} \tag{58}$$

where we use the following shorthand:

$$\mathbb{E}\left[m_{i,kl}^\bullet\right] = \mathbb{E}\left[m_i^\bullet \mid \delta_{i,k}^\circ = \delta_{i,l}^\bullet = 1\right] \quad \forall \quad k, l \in \{\boldsymbol{a}, \boldsymbol{b}\}.$$

Bus → Bus Consider first the possibility "***bb***," where i chooses bus in both scenarios, and the associated term $c_{\boldsymbol{bb}}$. In this case, the compensated expenditure level is a constant:

$$\mathbb{E}\left[m_{i,\boldsymbol{bb}}^\bullet\right] = \mu_{i,\boldsymbol{bb}}^\bullet = y_i - \psi_i^\bullet \tag{59}$$

Here, i's utility will not be affected by the changes in travel time or costs, but may increase due to the toll revenue refund payment, so the compensation level should exactly negate this refund payment; in other words, the expenditure level required to maintain the same utility level is a single quantity, and moreover, it is equal to the initial level minus the refund, $y_i - \psi_i^\bullet$.

As to the probability that "***bb***" is actually the case for individual i, we use a probability function P^* that is dependent on the compensated expenditure level, y_i:

$$P_{i,\boldsymbol{b}}^*\left(\mu_{i,\boldsymbol{bb}}^\bullet\right) = P_{i,\boldsymbol{b}}^*\left(y_i - \psi_i^\bullet\right) = \frac{e^{v\left(y_i - p_{\boldsymbol{b}}^\circ, t_{\boldsymbol{b}}^\circ\right)}}{e^{v\left(y_i - p_{\boldsymbol{b}}^\circ, t_{\boldsymbol{b}}^\circ\right)} + e^{\max\left[v\left(y_i - p_{\boldsymbol{a}}^\circ, t_{\boldsymbol{a}}^\circ\right), v\left(y_i - p_{\boldsymbol{a}}^\bullet, t_{\boldsymbol{a}}^\bullet\right)\right]}} \tag{60}$$

To illustrate this formulation, first consider that the individual would choose bus both before and after the change (with compensation) if two conditions are satisfied: 1) the individual's maximum utility is attained by the bus option under the uncompensated initial conditions, and 2) the individual would not change to the auto option, even if compensated to some expenditure level $\mu^{\bullet}_{i,\boldsymbol{bb}}$—in other words, for a counter-hypothetical switch from bus to auto, $\mu^{\bullet}_{i,\boldsymbol{bb}}$ would not suffice to maintain the original level of utility. This second criterion uses a constructed choice situation, which serves not to simulate any realistic choice situation, but rather to test whether the compensation level is sufficient to restore the original utility level.

The joint probability of these two conditions being satisfied can be expressed as:

$$\begin{aligned} P^{*}_{i,\boldsymbol{b}}\left(\mu^{\bullet}_{i,\boldsymbol{bb}}\right) = \Pr\Big\{ & v\left(y_i - p^{\circ}_{\boldsymbol{b}}, t^{\circ}_{\boldsymbol{b}}\right) + \varepsilon_{i,\boldsymbol{b}} \geq v\left(y_i - p^{\circ}_{\boldsymbol{a}}, t^{\circ}_{\boldsymbol{a}}\right) + \varepsilon_{i,\boldsymbol{a}} \\ & \bigcap v\left(y_i - p^{\circ}_{\boldsymbol{b}}, t^{\circ}_{\boldsymbol{b}}\right) + \varepsilon_{i,\boldsymbol{b}} \geq v\left(\mu^{\bullet}_{i,\boldsymbol{bb}} + \psi^{\bullet}_i - p^{\bullet}_{\boldsymbol{a}}, t^{\bullet}_{\boldsymbol{b}}\right) + \varepsilon_{i,\boldsymbol{a}}\Big\} \end{aligned} \tag{61}$$

which we can simplify to:

$$\begin{aligned} P^{*}_{i,\boldsymbol{b}}\left(\mu^{\bullet}_{i,\boldsymbol{bb}}\right) = \Pr\Big\{ & v\left(y_i - p^{\circ}_{\boldsymbol{b}}, t^{\circ}_{\boldsymbol{b}}\right) + \varepsilon_{i,\boldsymbol{b}} \\ & \geq \max\left[v\left(y_i - p^{\circ}_{\boldsymbol{a}}, t^{\circ}_{\boldsymbol{a}}\right) + \varepsilon_{i,\boldsymbol{a}}, v\left(\mu^{\bullet}_{i,\boldsymbol{bb}} + \psi^{\bullet}_i - p^{\bullet}_{\boldsymbol{a}}, t^{\bullet}_{\boldsymbol{a}}\right) + \varepsilon_{i,\boldsymbol{a}}\right]\Big\}. \end{aligned} \tag{62}$$

We know some things that can help us express (62) more precisely. First, our case is binomial logit, so we can restate this joint probability as:

$$P^{*}_{i,\boldsymbol{b}}\left(\mu^{\bullet}_{i,\boldsymbol{bb}}\right) = \frac{e^{v\left(y_i - p^{\circ}_{\boldsymbol{b}}, t^{\circ}_{\boldsymbol{b}}\right)}}{e^{v\left(y_i - p^{\circ}_{\boldsymbol{b}}, t^{\circ}_{\boldsymbol{b}}\right)} + e^{\max\left[v\left(y_i - p^{\circ}_{\boldsymbol{a}}, t^{\circ}_{\boldsymbol{a}}\right), v\left(\mu^{\bullet}_{i,\boldsymbol{bb}} + \psi^{\bullet}_i - p^{\bullet}_{\boldsymbol{a}}, t^{\bullet}_{\boldsymbol{a}}\right)\right]}} \tag{63}$$

Second, the compensated expenditure level in case "***bb***" is simply the individual's original income minus the refund, $y_i - \psi^{\bullet}_i$, as shown in (59). Substituting this for the compensated expenditure level $\mu^{\bullet}_{i,\boldsymbol{bb}}$, we arrive at the probability function given in (60).

Auto → Auto Next, consider the possibility "***aa***", where auto is chosen both before the policy the change, and afterwards with compensation. For these individuals, costs rise and travel times decrease outright, and the compensation level is computed using the systematic utility function to find the expenditure level that equates the new utility level to the original utility level, which we express using the function $\mu(\cdot)$:

$$\begin{aligned} \mathbb{E}\left[m^{\bullet}_{i,\boldsymbol{aa}}\right] = \mu^{\bullet}_{i,\boldsymbol{aa}} &= \mu\left(y_i - p^{\circ}_{\boldsymbol{a}}, t^{\circ}_{\boldsymbol{a}}; \psi^{\bullet}_i - p^{\bullet}_{\boldsymbol{a}}, t^{\bullet}_{\boldsymbol{a}}\right) \\ &= \arg\left[\mu : v\left(y_i - p^{\circ}_{\boldsymbol{a}}, t^{\circ}_{\boldsymbol{a}}\right) = v\left(\mu + \psi^{\bullet}_i - p^{\bullet}_{\boldsymbol{a}}, t^{\bullet}_{\boldsymbol{a}}\right)\right]. \end{aligned} \tag{64}$$

Unlike in the definition of the CV in (26), here we can use the observable utility function, $v(\cdot)$, which does not require the unobservable portion of utility to be known. This is because we are considering only the circumstance where *i*'s mode choices are already known to be auto in both cases, and for a single mode, the unobservable component is identical in both scenarios: $\varepsilon^{\circ}_{i,\boldsymbol{a}} = \varepsilon^{\bullet}_{i,\boldsymbol{a}}$. Consequently, the expenditure level in (64) is straightforward to compute, given the systematic utility function.

Note, however, that it is not clear outright whether $\mu^{\bullet}_{i,\boldsymbol{aa}} + \psi^{\bullet}_i$ will be greater or less than the original expenditure level, y_i. If the cost increase outweighs the travel time reduction and refund payment in utility terms, then a positive compensation will be needed, so we will have $\mu^{\bullet}_{i,\boldsymbol{aa}} > y_i - \psi^{\bullet}_i$; if the opposite is the case, then a negative compensation will be needed to restore the original utility level, and $\mu^{\bullet}_{i,\boldsymbol{aa}} < y_i - \psi^{\bullet}_i$.

For the probability of individual i realizing the possibility "***aa***", we can again use the probability given in (63), except that the subscripts for auto and bus are reversed, and we evaluate the probability at the compensated expenditure level $\mu^{\bullet}_{i,\boldsymbol{aa}}$:

$$P^*_{i,\boldsymbol{a}}\left(\mu^{\bullet}_{i,\boldsymbol{aa}}\right) = \frac{e^{v\left(y^\circ_i - p^\circ_{\boldsymbol{a}}, t^\circ_{\boldsymbol{a}}\right)}}{e^{v\left(y^\circ_i - p^\circ_{\boldsymbol{a}}, t^\circ_{\boldsymbol{a}}\right)} + e^{\max\left[v\left(y^\circ_i - p^\circ_{\boldsymbol{b}}, t^\circ_{\boldsymbol{b}}\right), v\left(\mu^{\bullet}_{i,\boldsymbol{aa}} + \psi^{\bullet}_i - p^{\bullet}_{\boldsymbol{b}}, t^{\bullet}_{\boldsymbol{b}}\right)\right]}}. \tag{65}$$

For a general form of the above joint probability in the binomial case, we can say:

$$P^*_{i,k}(\mu) = \frac{e^{v\left(y^\circ_i - p^\circ_k, t^\circ_k\right)}}{e^{v\left(y^\circ_i - p^\circ_k, t^\circ_k\right)} + e^{\max\left[v\left(y^\circ_i - p^\circ_l, t^\circ_l\right), v\left(\mu + \psi^{\bullet}_i - p^{\bullet}_l, t^{\bullet}_l\right)\right]}} \quad \forall \quad k, l \in \{\boldsymbol{a}, \boldsymbol{b}\}, k \neq l. \tag{66}$$

Auto → Bus Next we consider the possibility "***ab***", that individual i chooses auto initially, then switches to bus when the prices and travel times change and a refund check is added, even with compensation. Here, the expected compensation and the probability of this being the case are jointly expressed as:

$$c_{\boldsymbol{ab}} = \begin{cases} -\int_{y_i - \psi^{\bullet}_i}^{\mu^{\bullet}_{i,\boldsymbol{aa}}} \mu \frac{\partial P^*_{i,\boldsymbol{a}}(\mu)}{\partial \mu} d\mu = \int_{y_i - \psi^{\bullet}_i}^{\mu^{\bullet}_{i,\boldsymbol{aa}}} \mu \, dP^*_{i,\boldsymbol{a}}(\mu) & \text{if } \mu^{\bullet}_{i,\boldsymbol{aa}} > y_i - \psi^{\bullet}_i, \\ 0 & \text{otw.} \end{cases} \tag{67}$$

To illustrate, first note that if this individual switched from auto to bus, then it must be the case that the cost increase outweighs the travel time reduction and refund payment in utility terms, hence we know that:

$$\mu^{\bullet}_{i,\boldsymbol{aa}} + \psi^{\bullet}_i > y_i \quad \Rightarrow \quad \mu^{\bullet}_{i,\boldsymbol{aa}} > y_i - \psi^{\bullet}_i. \tag{68}$$

Also in the case "***ab***", we cannot say that the compensated expenditure level will be a single value, since it depends on the unobserved component $\boldsymbol{\varepsilon}_i$. However, we can at least say that it is bounded above by $\mu^{\bullet}_{i,\boldsymbol{aa}}$, since in this case, the decision-maker could at least improve utility somewhat by switching modes, so compensation fully at the level of "***aa***" is unnecessary. We can also say that some positive compensation will be necessary, since the individual has switched to a new alternative, bus, that was not originally the preferred alternative, auto, and the newly chosen bus alternative has not improved at all in either travel times and costs. The new expenditure level plus refund payment is therefore bounded below by y_i, so the expenditure level alone must be bounded below by $y_i - \psi^{\bullet}_i$.

For possibility "***ab***", instead of taking a fixed compensation level and the probability of that level, we take the range from $y_i - \psi^{\bullet}_i$ to $\mu^{\bullet}_{i,\boldsymbol{aa}}$ and integrate across their probabilities using the joint probability function $P^*_{i,\boldsymbol{a}}$ given in (63). This probability has negative slope; consider that as the incentive (i.e. the compensated expenditure level) to switch modes increases, the probability of remaining with the original mode decreases. The complementary outcome is the individual who *does* switch, for which we use the complementary probability $1 - P^*_{i,\boldsymbol{a}}$, given the original choice of auto.

Bus → Auto Finally, there is the possibility "***ba***", that the decision-maker would start out taking bus, then switch to auto. The joint expectation and probability term here is given by:

$$c_{\boldsymbol{ba}} = \begin{cases} -\int_{\mu^{\bullet}_{i,\boldsymbol{aa}}}^{y_i - \psi^{\bullet}_i} \mu \frac{\partial P^*_{i,\boldsymbol{b}}(\mu)}{\partial \mu} d\mu = \int_{\mu^{\bullet}_{i,\boldsymbol{aa}}}^{y_i - \psi^{\bullet}_i} \mu \, dP^*_{i,\boldsymbol{b}}(\mu) & \text{if } \mu^{\bullet}_{i,\boldsymbol{aa}} < y_i - \psi^{\bullet}_i, \\ 0 & \text{otw.} \end{cases} \tag{69}$$

Note that for this possibility to be realized requires that the travel time reduction and refund payment outweigh the cost increase in utility terms, meaning that $\mu^{\bullet}_{i,\boldsymbol{aa}} < y_i - \psi^{\bullet}_i$, which is the complement of the requirement for "***ab***". Without compensation, a change from bus to auto would need to cause an improvement in utility for such a decision to be actually made. The compensation, then, would be negative, set exactly to bring the individual's utility back to the original level; hence the compensated expenditure level, with the refund payment, would need to be less than the expenditure level y_i, so the compensated budget, without the refund payment, would be bounded above by $y_i - \psi^{\bullet}_i$. Moreover, the lower bound of the expenditure level here is given by $\mu^{\bullet}_{i,\boldsymbol{aa}}$, since at that level, the decision-maker would not have a sufficient expenditure level with the compensated auto alternative to restore the original utility level. To quantify the expected expenditure level for possibility "***ba***", we use an analogous formulation to (67), but with modified bounds as described above, resulting in the expression in (69).

Altogether, the expected compensated expenditure level for individual i, unconditional of mode choice before and after, is:

$$\begin{aligned}\mathbb{E}\left[m^{\bullet}_i\right] &= c_{\boldsymbol{bb}} + c_{\boldsymbol{aa}} + c_{\boldsymbol{ab}} + c_{\boldsymbol{ba}} \\ &= \mu^{\bullet}_{i,\boldsymbol{aa}} P^{*}_{i,\boldsymbol{a}}\left(\mu^{\bullet}_{i,\boldsymbol{aa}}\right) + \left(y_i - \psi^{\bullet}_i\right) P^{*}_{i,\boldsymbol{b}}\left(y_i - \psi^{\bullet}_i\right) \\ &\quad - \begin{cases} \int_{y_i - \psi^{\bullet}_i}^{\mu^{\bullet}_{i,\boldsymbol{aa}}} \mu \, dP^{*}_{i,\boldsymbol{a}}(\mu) & \text{if } \mu^{\bullet}_{i,\boldsymbol{aa}} > y_i - \psi^{\bullet}_i, \\ \int_{\mu^{\bullet}_{i,\boldsymbol{aa}}}^{y_i - \psi^{\bullet}_i} \mu \, dP^{*}_{i,\boldsymbol{b}}(\mu) & \text{if } \mu^{\bullet}_{i,\boldsymbol{aa}} < y_i - \psi^{\bullet}_i, \\ 0 & \text{otw}, \end{cases}\end{aligned} \tag{70}$$

where $\mu^{\bullet}_{i,\boldsymbol{aa}}$ is given by (64), and the probabilities $P^{*}_{i,\boldsymbol{a}}$ and $P^{*}_{i,\boldsymbol{b}}$ are given by (66). Note that only one of the integral terms will remain, depending on whether $\mu^{\bullet}_{i,\boldsymbol{aa}}$ is greater or less than the original expenditure level (i.e. income) minus the refund payment, $y_i - \psi^{\bullet}_i$; this determination, in turn, depends on the specification of the utility function and the values its coefficients, as well as on the amount of the refund. In the general form given by (31), this selection of integral terms is accomplished by identifying the minimum compensated expenditure level by (33) and using it as the lower bound of the integrals.